Particle-gas dynamics in the protoplanetary nebula

Jeffrey N. Cuzzi (Ames Research Center)

Joelle M. Champney (A. T. M. inc.), Anthony R. Dobrovolskis (U. C. Santa Cruz)

In the past year we have made significant progress in improving our fundamental understanding of the physics of this problem, as detailed below. Furthermore, having brought our code to a state of fairly robust functionality, we devoted significant effort to optimizing it for running long cases. We optimized the code for vectorization to the extent that it now runs eight times faster than before (a typical case used to take a substantial fraction of a Cray 2 hour to run to convergence).

Physical improvements to the model

In starting to model very dense particle layers, when the particle mass density can exceed 100 times the local gas mass density, we realized that in such regions, the viscosity arising from interparticle collisions may become comparable to, or even exceed, the turbulent gas viscosity, and began to explore realistic implementations of particle viscosity ν_p. One simple parametrization of ν_p is the particle viscosity routinely used in planetary rings which are not overly optically thick (*e.g.* Goldreich and Tremaine 1978, Wisdom and Tremaine 1988):

$$\nu_p = (\frac{v_p^2}{2\Omega})\frac{f}{1+f^2}, \tag{1}$$

where v_p is the average particle random velocity, Ω is the orbital frequency, and f is the vertically integrated particle area filling factor or optical depth. The trick is to estimate v_p. A simple ring-type assumption such as $v_p = \Omega\delta$, where δ is the particle layer thickness, is inappropriate in this case since *global* particle motions on the scales δ and Ω may be driven by turbulent eddies without *nearby* particles having much relative velocity at all if they are sufficiently well coupled to the local gas velocity.

We have come up with a simple model for ν_p based on the particle stopping time t_p, which determines the Schmidt number Sc. The model begins with the scaling relationships

$$\nu_p \sim <v><l> \sim <\omega><l>^2 \sim <v>^2 / <\omega>, \tag{2}$$

where $<v>, <l>,$ and $<\omega>$ are the characteristic relative velocity, length scale, and collision frequency of the particles respectively. There are two regimes of interest. If the collision time and length scales are not limited by the global scales of the system (layer thickness δ and orbital frequency Ω), then $\omega \sim n\pi r^2 v_{rel} \sim v_{rel}/l^*$, where l^* is the mean free path between collisions. From the work of Völk et al (1980), we identify v_{rel} as a fraction $\sqrt{t_p/t_g} \leq 1$ of the global average particle random velocity v_p when $t_p \leq t_g$, where t_p is the particle stopping time and t_g is the eddy turnover time. Naturally, v_{rel} can never exceed v_p, and we account for this in the limit $t_p >> t_g$. We also note from our Schmidt number model (cf. also Völk et al 1980) that $(v_g/v_p)^2 = Sc = (1 + t_p/t_g)$, or $v_p \sim v_g Sc^{-\frac{1}{2}}$. Consequently, $\nu_p = l^* v_{rel} = l^* v_p \mathcal{F}(Sc)$, where

$$\mathcal{F}(Sc) = \frac{\sqrt{Sc-1}}{\sqrt{Sc}} \text{ if } t_p < t_g, \text{ and } \mathcal{F}(Sc) = \frac{1}{\sqrt{Sc}} \text{ if } t_p > t_g. \tag{3}$$

Note that this implies the particle viscosity goes to zero for very small t_p regardless of the particle density. This is because the particles are all trapped to the same local gas velocity and have no relative random velocity at all.

In another limit, if the scales of the system limit the collision frequency or mean free path (as in, for instance, optically thin planetary ring systems), $\nu_p \sim f < v >< l >$ or with the same assumptions as above, $\nu_p = f\nu_t/Sc$ where ν_t is the turbulent gas viscosity.

We have implemented viscous terms in our numerical code using these parametrizations of ν_p. For the cases we have been studying (30 - 100 cm radius particles, 1 and 10 AU, minimum mass solar nebula) the particle optical depth f is on the order of 0.1 and the Schmidt number is on the order of unity; consequently the particle viscosity is about 10% of the gas turbulent viscosity.

Numerical results

Using the newly vectorized code, we ran several models which included particle viscosity terms both at 1 AU and at 10 AU. The code is well behaved in both limits. The results differ from previous runs in that mean radial and azimuthal velocities in the particle layer are now more slowly varying with vertical distance from the midplane due to the increased coupling by particle viscosity.

Reynolds averaging of fluid equations

One aspect of our model that we wanted to put on firmer ground is our mixed use of Favre (mass) averaging for the momentum equations with Reynolds (time) averaging for the particle conservation equation. It is the Reynolds averaging that results in the diffusion term we use to model particle layer diffusion, whereas similar terms are suppressed in the Favre averaged equations. Feeling that this was rigorously inconsistent and perhaps even quantitatively important, we have devoted considerable effort to rewriting the momentum equations from the Reynolds-average standpoint. At this time we have obtained the new correlation terms but not as yet coded them up. They are all tractable and can be modeled in very much the same way as the gas eddy viscosity is always derived as a model of the Reynolds stresses, and heat transport and particle diffusion by convection are modeled with the gradient diffusion hypothesis (using the Prandtl and Schmidt numbers respectively). The terms are small, and we expect them to change our numerical results in minor but potentially very interesting ways.

Modeling of turbulence and viscosity

In our prior work, we have explored two independent parametrizations of the nebula turbulence, of differing complexity. In Champney and Cuzzi (1990), we pointed out the poorer than desirable agreement between the eddy viscosity as determined from the two-equation $(k - \epsilon)$ model and that obtained with our current Prandtl model, which is characterized by only one parameter (the critical Reynolds number). Since the two-equation model is partly *ad hoc* and contains at least five constants, we have chosen so far to use the simpler Prandtl model. However, as we pointed out last year, not even the Prandtl model is without its uncertainties. The critical Reynolds number Re^* (Champney and Cuzzi 1990, equation 49) depends on the nature of the flow regime. Heretofore we have used a value of 500 for Re^*, but now believe that the true value of Re^* is about 100. Use of this number brings the two-equation and Prandtl models into agreement.

However, the Prandtl technique cannot model the damping of turbulence by the particle phase (Sproull 1961, Elghobashi and Abou-Arab 1973, Pourahmadi and Humphrey 1983); this may be very important not only in the shear layer, but also in earlier phases of the nebula

428

when particle settling and accretion occurs in the presence of widespread convective turbulence. Consequently we are delving more deeply into self-consistent turbulence models.

The two-equation models currently in use (*e.g.* Rodi 1984) postulate one equation for the generation, transport, and damping of the turbulent kinetic energy k, and a similar equation for the energy dissipation rate ϵ. We have verified that the k-equation (including its particle damping terms) is derivable in a straightforward way from the basic fluid equations, while as far as we can determine, the ϵ-equation is a relatively *ad hoc* creation designed to improve fits to data. In fact, prior to the current widespread use of $k - \epsilon$ models, use of only the k-equation was standard (Rodi 1984). We find that the dissipation rates ϵ calculated with our two-equation model are approximated by k/t_g, where the eddy turnover time t_g is simply the inverse of the orbital frequency. In the coming year, we plan to replace the ϵ-equation entirely by the simple scaling $\epsilon = k/t_g$ in the k-equation, simplifying the method to a one-equation model. This will expedite the study of turbulence in the shear layer, including particle damping.

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